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The Pumping Capacity of Impellers in Stirred Tanks

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The flow characteristics produced by an impeller in a stirred tank must be known if the overall behavior of the stirred tank is to be predicted. Correlations of impeller power requirements are available for a wide variety of impeller and tank geometries (*1*), but there is relatively little quantitative information about impeller pumping capacities.

Rushton, Mack, and Everett (*4*) attempted the determination of pumping capacities in a special two-tank assembly. Water was pumped by a propeller through an orifice in the bottom of the smaller tank and into the larger surrounding tank. The geometry of the system was adjusted to give a maximum flow, and this maximum flow was assumed to be the actual propeller pumping capacity. These authors found the volumetric pumping rate (q) to be dependent upon the propeller speed (N) and propeller diameter (D_I) according to the equation

$$q = K N D_I^2 \qquad (1)$$

where K is a constant. Some additional data were obtained for turbine impellers with the same experimental technique.

A later paper (*5*) reports the volumetric pumping rate to be dependent on the cube of propeller diameter with the value of K given as 0.4 for water at 70°F.

Sachs and Rushton (*6*) describe a photographic technique that was employed to study both the pumping capacity and the overall circulation caused by turbine impellers. Van de Vusse (*8*) presents a theoretical discussion of the factors defining impeller pumping capacities.

The present paper discusses a new experimental technique for determining pumping capacities. This method is believed to be superior to those previously reported in that it is quite simple and does not disturb the system being studied. Furthermore analysis of the resulting data can give information about overall circulation rates as distinct from pure pumping capacities and point the way to the writing of a mathematical model describing batch mixing experiments and the transient behavior of continuous-flow stirred tanks (*3*).

THE FLOW-TIME DISTRIBUTION EXPERIMENT

The new procedure for studying pumping capacities in stirred tanks involves a flow-time distribution experiment in which the time required for fluid to flow around the circulation pattern of a stirred tank is measured. This measurement is made by watching a small object (called the *flow follower*) as it is carried about the tank and noting each time it passes some chosen reference point. Several hundred observations of this kind are made during one experiment.

Two important conditions must be met if the experimental results are to have a useful meaning. First the flow follower must have the same density as

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the tank fluid so that its velocity relative to the fluid is negligible. Second the reference point must be that region in the tank where the flow paths of interest begin and end.

The results obtained from this experimental procedure are the average time for travel in the circulation pattern and the distribution of flow times about the average.

THEORY

The basic theory underlying these experiments is given by Danckwerts (2) who shows that if a system of volume V is flushed with a flow rate q, regardless of the internal characteristics of the volume, the average time of travel through the volume is given by V/q.

Thus in the present study, where observations were made of the flow follower as it passed through the cross section of a marine propeller, the average time for flow around the circulation pattern of the tank was related to the tank volume and the pure propeller pumping capacity by the expression

$$T_{av} = V_T/q_I \qquad (2)$$

The theoretical discussion of the distributions of flow times found by this method is somewhat more involved.

Visual observation of the circulation patterns in propeller-driven stirred tanks, as well as observations made during batch-mixing-time experiments (3), show that the flow is quite complex. There are apparently two flow rates of interest: the pure pumping capacity and the flow induced by the propeller discharge jet.

It is reasoned that the induced flow is generated and maintained by momentum transport between the propeller discharge jet and the fluid surrounding the jet. The fluid of this secondary flow moves in a doughnut pattern, never, by definition, going through the propeller.

Fluid pumped by the propeller travels in a fairly well-defined jet to the bottom of the tank, then outward and up the walls of the tank. At the level of the propeller this primary flow enters what may be called the *upper volume* of the tank. Here there is an infinite number of paths which elements of fluid may travel in returning to the propeller, so that the time required for travel through the upper volume is distributed about a mean time T_2.

The authors propose two ideas as a result of these observations. First the average residence time measured using the propeller as a reference zone is characteristic of the pure pumping capacity alone and contains no contribution from the induced flow. That is to say Equation (2) is the correct interpretation of T_{av} data reported below. This fact may be proved for any combination of flow rates and artificial volumes, if it is assumed that the probability of the flow follower entering a volume is proportional to the flow through that volume.

A second hypothesis deals with the distributions of flow times about the average value. It seems reasonable to say that there are three separate volumes in a propeller-driven stirred tank. One contains the induced flow and is of no interest here, but the other two are the volume beneath the propeller (V_1) through which the total pumped flow travels and the volume above the propeller. Therefore it can be said that the average flow time is the sum of several terms related to the time required for travel through each of these volumes.

PROCEDURE

Experiments were carried out in a 12 in. O.D., 11.5 in. I.D., flat-bottom glass tank using an experimental agitator mounted on a wooden support frame. Liquid holdup was determined by direct measurement with a meter stick. The propellers used were three-blade, square pitch modified marine propellers with a pitch ratio of 1.0. Water was the tank fluid in all runs, and only the turbulent regime of operation was studied.

Data were obtained by watching a small sheet of rubber with the approximate dimensions 1/4 × 1/4 × 1/16 in. and having a specific gravity of about 0.98 as it was carried about the tank in the circulation pattern set up by the propeller. Each time this flow follower passed through the horizontal cross section of the propeller a number was spoken into a tape recorder. After about 200 such countings had been made, the time interval between consecutive numbers was measured (in recorder playback) with a stopwatch. Data of this type were obtained for the following ranges of variables.

Propeller diameter, in.	D_I	2½ to 5
Propeller, rev./min.	N	300 to 900
Tank volume, cu. in.	V_T	919 to 1267
Average circulation time, sec.	T_{av}	1.30 to 8.43

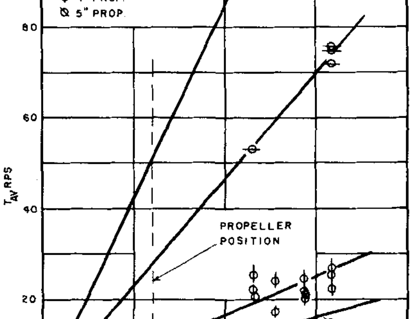

Fig. 1. Flow-time data for propellers. The liquid volume below the propeller is indicated by the dashed line.

RESULTS

Figure 1 shows most of the data obtained for this report; a propeller speed of 700 rev./min. for both the 5- and 2½-in. propellers defines the range of Reynolds numbers represented. At any tank volume the product NT_{av} is independent of the propeller speed N, so that the propeller pumping capacity must vary linearly with speed. It follows from Equation (2) that

$$q_I = K'N \qquad (3)$$

where K' is a constant dependent on propeller diameter. From previous findings (5, 8) K' is taken to be

$$K' = KD_I^3 \qquad (4)$$

Analysis of the data for twenty-three runs gives the result

$$NT_{av}/V_T = 1/(0.70\,D_I^3) \qquad (5)$$

so that

$$q_I = 0.70\,ND_I^3 \qquad (6)$$

The average deviation of K from the mean of 0.70 is 13.5%, and the maximum deviation is 26.5%.

Figure 2 shows that nature of the distribution of residence times about the average. It is evident that the distribution is made up of two major parts. The first of these, a pure time delay, represents the minimum time required for flow from the propeller back to the propeller. The volume associated with this flow time (T_1) has been designated V_1. If it is assumed that the fluid moves through this region in plug flow at the rate q_I, then

$$V_1 = q_I T_1 \qquad (7)$$

and the size of this volume can be estimated. For the systems described in this paper V_1 averages about 42% of the total volume below the propeller.

The remaining part of the distributions of Figure 2 shows how flow time can vary owing to various linear fluid velocities and path lengths. No analysis of this part of the distributions can be given here, since a knowledge of the flow induced by the propeller discharge jet is necessary but as yet unavailable. Reference 7 represents one of two studies which are more complete in this respect; these will be reported on at a later time.

DISCUSSION

The flow-time distribution experiment will give truly representative results only when the flow follower travels in representative flow paths. This condition can be violated if the liquid depth above the propeller is too large, for then the flow follower encounters regions of essentially zero linear velocity wherein its motion is governed more by gravitational forces than drag forces. Sharp changes of flow pattern have been noted (3) to reduce the size of the active, well-flushed volume of such a tank.

In these preliminary studies of propeller pumping it appears that too few observations of flow time were taken, so that runs made with large tank volumes do not represent the true picture. The recorded observations give too much weight to the active volume, not enough to the inactive, and calculations of the pumping capacity from these data are in error because the total tank volume is larger than the effective volume related to the observed average flow times.

Closer scrutiny of the data shows the magnitude of this volume effect. For the runs made at volumes of 1,267, 1,144, and 1,021 cu. in. the K values are 0.75, 0.86, and 0.80 respectively. Batch mixing experiments (3) indicate that the total tank volume is well flushed at a volume of 919 cu. in., and the average K found from runs at that volume is 0.61 with an average deviation of less than 8% and a maximum deviation of 15%.

Thus it is proposed that the pure pumping capacity of the marine propellers used in this work is described by the equation

$$q_I = 0.61\ ND_I^3 \qquad (8)$$

and that this relation is not dependent on the propeller Reynolds number in the turbulent regime of operation. Such a finding implies that K is independent of fluid viscosity, but that the power required for a given pumping job is proportional to the fluid density.

It is possible to estimate that fraction of the power input to a stirred tank which is used for pumping in the following way. Assume that a propeller accelerates the fluid it pumps from a zero velocity up to a discharge velocity given by

$$v_I = 4q_I/\pi D_I^2 \qquad (9)$$

The kinetic energy of a unit volume of the propeller discharge fluid is, therefore

$$K.E. = \rho v_I^2/2g_c = 8K^2\rho N^2D_I^2/g_c\pi^2 \qquad (10)$$

and the power required to pump fluid at the rate q_I is

$$P = 8K^3\rho N^3D_I^5/g_c\pi^2 \qquad (11)$$

The value of the power number for a system of this type in the turbulent regime of operation is known (*1*) to be about 0.3. Thus

$$P = 0.3\epsilon\rho N^3D_I^5/g_c = 8K^3\rho N^3D_I^5/g_c\pi^2 \qquad (12)$$

where ϵ is the fraction of the power input that is used for pumping, and the first equality is an expression of the power number. The solution of Equation (12) yields

$$\epsilon = 8K^3/0.3\pi^2 = 0.62 \qquad (13)$$

While the error in this estimate may be large, it does seem reasonable to conclude that only 50 to 75% of the power input to a propeller goes to pumping. The rest of the power appears initially as turbulent energy in the propeller discharge jet and is used in maintaining the radial gradients of

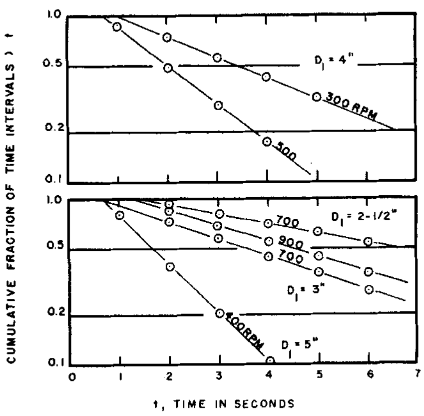

Fig. 2. Flow-time distribution data for various propeller diameters and speeds. Propellers positioned 4.7 in. from tank bottom. Tank volume: 919 cu. in. Baffle width: 1½ in.

angular velocity which necessarily exist in the tank. Eventually of course all of the input power is dissipated by the momentum transport which maintains the flow pattern of the tank fluid.

But this discussion of power requirements and power use is pertinent for other reasons. It shows that the discrimination used in selecting the best value of K as 0.61 may be justified on the basis of a knowledge of power requirements.

Specifically it is known that the power number does not depend on liquid depth or propeller height over reasonable ranges of these variables (*1*). When it is assumed that power and pumping are always proportional as liquid depth and propeller positiôn are varied, it follows that the pumping capacity of a propeller also does not change with tank volume and that the discrimination used above is allowable.

CONCLUSIONS

Flow-time distribution experiments are unique in that they are simply executed and simply analyzed. The flow pattern and pumping capacity of an impeller in any tank may be determined in a few hours if a flow follower can be seen moving about the tank. The fact that the experimental method does not disturb the system being studied is of particular importance.

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NOTATION

D = diameter
g_c = Newton's law conversion factor, 32.2 poundals/lb.
K = constant
K' = constant
N = rotational speed of impeller
P = power
q = volumetric flow rate
T = time constant or average time
t = time
V = volume
v = velocity
ϵ = fraction of total power used for pumping
ρ = fluid density

Subscripts

av = average
I = impeller
T = tank or total
1 = volume below the propeller
2 = volume above the propeller

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